Linearization and Differentials

Linearization

Definition. *Let $f(x)$ be a differentiable function with domain D, a is a real number in D. We define* **the Linerization of $f(x)$ at $x = a$** *to be*

$$L(x) = f(x) + f'(a)(x - a)$$

Example 1. *Use the linearization of $f(x) = x^2$ at $x = 1$ to estimate 1.01^2.*

The first derivetive of $f(x)$ is $f'(x) = 2x$. The linearization of $f(x) = x^2$ at $x = 1$ is

$$L(x) = f(1) + f'(1)(x - 1) = 1 + 2(x - 1)$$

Hence 1.01^2 is approximated by $L(1.01) = 1 + 2(1.01 - 1) = 1.02$.

Differential

Definition. *Let $y = f(x)$ be a differentiable function, the* **differential** *of $y = f(x)$ is defined as*

$$dy = f'(x)dx$$

Example 2. *Let $y = f(x) = x^2$, find its differential.*

The first derivetive of $f(x)$ is $f'(x) = 2x$. The differential is

$$dy = 2xdx$$